Advanced operations research Name: Time: 120 min

1. Consider the problem

$$max \ \ |x_1 - 3x_2|$$
$$|x_1 + 2| + |x_2| \leq 5.$$

How we can solve this problem using linear programming.

2. Consider a school district with I neighborhoods, J schools, and G grades at each school. Each school j has a capacity of C_{jg} for grade g. In each neighborhood i, the student population of grade i is S_{ig} . Finally, the distance of school j from neighborhood i is d_{ij} . Formulate a linear programming problem whose objective is to assign all students to schools, while minimizing the total distance travelled by all students. (You may ignore the fact that numbers of students must be integer.)

3. If S is an open set, show that the problem $\min_{x \in S} \ cx$ where $c \neq 0$ possesses no optimal solution.

4. Find the extreme directions of the following problem:

$$-x_1 + x_2 = 4$$
$$x_1 - 2x_2 + x_3 < 6$$
$$x_3 \geq 1$$
$$x_1, x_2, x_3 \geq 0.$$

5. Consider the following polyhedral set:

$$X = \{x| - 3x_1 + x_2 \leq -2, \ - x_1 + x_2 \leq 2, \ - x_1 + 2x_2 \leq 8, \ - x_2 \leq -2, \ x_1, x_2 \geq 0\}.$$

Represent $\bar{x} = (4, 3)$ as a convex combination of the extreme points and a nonnegative combination of the extreme directions.